**1**

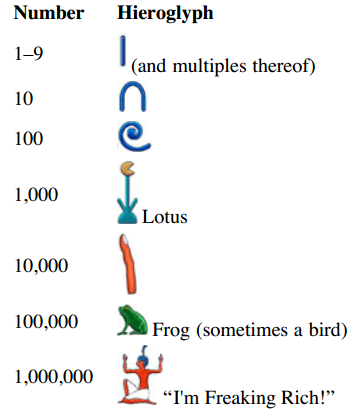
**CHAPTER**

**Number System**

A *number system* is a way in which humans represent numbers. Because human among all known species has ability to count and form numbers which later can perform calculations upon. Among the different civilizations and cultures throughout the ages, and there still exists a wide variety of numbers today, in our comparatively global society. Actually humans didn't invent numbers, just the symbols for them.

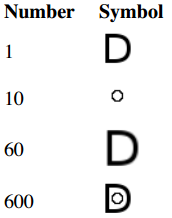
*Number sets* are sets of numbers that we are able to perform mathematical operations upon. As we will not discuss them further, we give some examples of them right here and rid ourselves of them: natural numbers (ℕ), real numbers (ℝ), rational numbers (ℚ), algebraic numbers, complex numbers , so on.

Soon after language develops, it is safe to assume that humans begin counting - and that fingers and thumbs provide nature's [**abacus**](http://www.historyworld.net/wrldhis/PlainTextHistories.asp?gtrack=pthc&ParagraphID=bxj#bxj). The decimal system is no ancient. Ten has been the basis of most counting systems in history. When any sort of record is needed, notches in a stick or a stone are the natural solution. In the earliest surviving traces of a counting system, numbers are built up with a repeated sign for each group of 10 followed by another repeated sign for 1.

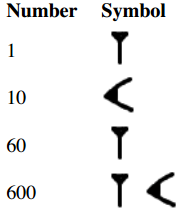
**Egyptian numbers: 3000-1600 B.C.** A system that uses symbols such as frog, heel bone, and lotus flower... to represent numbers. It is easy to understand, but calculation can become quickly inconvenient. It is a base 10 system, thus it is a decimal system. The hieroglyphic form uses symbols for powers of ten, based on repetition for other numbers. As the Egyptians wrote from right to left, the largest power of ten appears to the right of the other numerals.

**The Sumerian and Babylonian numbers: 1750 B.C.** The Sumerians with their base 60 and their place-value system, what marvels of human ability. The Summerians lived in the regions of the lower Tigris and Euphrates valleys around 3000 B.C. Later on, as the importance of the city of Babylon grew, power was shifted upstream and the Sumerians more or less became the Babylonians. It uses only two symbols to represent all possible numbers. However, it is probably one of the most doubtful of all numerals.

**Sumerians Numerals**

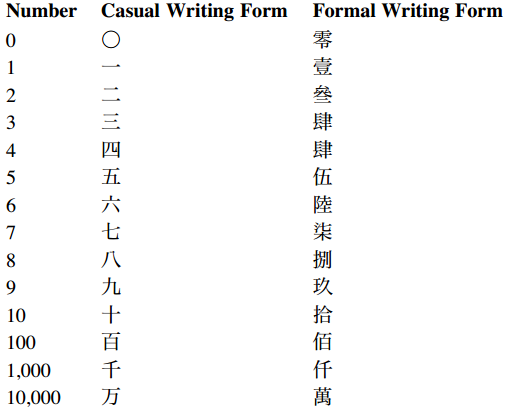


**Babylonian Numerals**



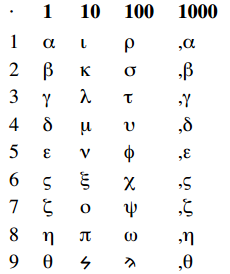
**The Chinese Number System 2500 B.C.**

Chinese has number words for 1 through 9 and for various powers of ten, much like English. Their way of forming larger numbers is also much the same as it is done in English, where each number is attached to a given multiple of ten, e.g., hundred, thousand, and so on. The system is, however, not a positional system, so words can be spoken in any order without changing the meaning of the number as a whole. Basically, used small bamboo rods arranged to represent the **numbers** 1 to 9, which were then places in columns representing units, tens, hundreds, thousands, etc.



**The Greeks Number System 500 B.C.**

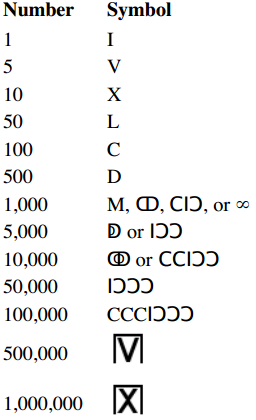
The Greek number system is known as the Ionian number system, which was the most beautiful number system. It is a decimal number system and comes with two notations, one older and a newer one using twenty-seven symbols  
of the alphabet, which was actually only contained of twenty-four symbols, so three were selected  
from an older alphabet. Here, we discuss the newer form, as the old one is much like that of the  
Romans, albeit even the old one is more advanced than the Roman one and they ruled Europe for how long?



**The Roman Number System**

Many of us have become familiar to this numerals system through the manipulation of books since many books use roman numerals as page numbers. However, explore it a little bit deeper here!

Not much of algebraic interest can be said about the Romans, other than that they were great in numbers and controlled a vast empire for hundreds of years, which at least counts for something. They did however have a beautiful way of writing numbers. Yet a highly artistic to write the Roman number system in some sense, as they used letters of the alphabet to form numbers. Anyway, the Roman number system came into use at the time around the beginning of the Common Era. Furthermore, a common misconception is that the Romans did not have numerals beyond one thousand. They did, they actually  
had a special symbol for multiples of 100,000, though it only came into use in a later stage of the  
civilization.



**The Modern Number System**

The Hindu-Arabic numerals were invented by mathematicians in India. Perso-Arabic mathematicians called them "Hindu numerals" (where "Hindu" meant Indian). Later they came to be called "Arabic numerals" in Europe, because they were introduced to the West by Arab merchants. We are quite easy with this modern number system and it is based 10 system which works fine. There is also a correlation between the number of fingers on our two hands and this system.

**Binary Number System**

Our own century has introduced another international number system, which most of us use but few are aware of it. This is the binary language of computers. When interpreting coded material by means of electricity, speed in tackling a simple task is easy to achieve and complexity merely complicates. So the simplest possible counting system is best, and this means one with the lowest possible base - 2 rather than 10.  
Instead of zero and 9 digits in the decimal system, the binary system only has zero and 1. So the binary equivalent of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 is 1, 10, 11, 100, 101, 110, 111, 1000, 1001 and 1010.

**Set of Integers:**

Set of integers is denoted by Z and defined as  or

**Rational Numbers:**

We know that a rational number is a number which can be put in form of, where.

The numbers etc. are rational numbers.

**Irrational Numbers**:

Irrational numbers are those numbers which cannot be written into the form  where .

The numbers  are irrational numbers.

**Terminating decimals Fraction:**

A decimal which has only a finite number of digits in its decimal part, is called a terminating decimal. Thus  are examples of terminating decimals.

i) 

are rational numbers.

**Recurring Decimals Fraction:**

This is another type of

rational numbers. In general, a recurring or periodic decimal is a decimal in which one or more digits repeat indefinitely.

i)  is a recurring decimal, so it is a rational number.

ii) is a rational number

iii) is a rational number.

**Theorem 1:**

**Prove that  is an irrational number.**

**Proof:** Assume contrarily that is a rational number so that it can be written in the form, where. Suppose further that is in its lowest form.

Then 

Squaring both sides we get;



 → (1)

The L.H.S. of this equation has a factor 2. This implies that R.H.S. must have the same factor.

This implies that *p* also has a factor 2…… (i),

then an integer *k* such that



Squaring both sides we get

 → (2)

From equations (1) and (2),





The R.H.S of this equation has a factor ‘2’ L.H.S must have the same factor.

Thehas a factor 2. This implies that *q* has also a factor 2……..(ii)

This implies that both *p* and *q* have factor 2, which contradicts the hypothesis that is in its lowest form. Hence  is an irrational number.

**Theorem 2:**  **Prove  is an irrational number.**

**Proof:** Assume contrarily that is a rational so that it can be written in the form, where. Suppose further that is in its lowest form.

Then 

Squaring both sides we get;



 → (1)

The L.H.S. of equation has a factor 3. This implies that R.H.S. must have the same factor.

This implies that *p* has also a factor 3, then an integer *k* such that



Squaring both sides we get

 → (2)

From equations (1) and (2),





has a factor 3. This implies that *q* has also a factor 3.

Since both *p* and *q* have factor 3, which contradicts the hypothesis that is in its lowest form. Hence  is an irrational number.

Similarly proof can be done for.

**Binary Operation:**

A binary operation in a set *S* is a rule that associates any elements *a* and *b* of *S* an element denoted by *a**b*. For all *a* and *b*, the elements *a**b* also belong to *S*. This is often taken to be implied in saying that is a binary operation on *S*.

**PROPERTIES OF REAL NUMBERS**

1. **Addition Properties**

**i) Closure Law of Addition**



**ii) Associative Law of Addition**



**iii) Additive Identity**





0 (read as zero) is called identity element of addition.

**iv) Additive Inverse**

 such that





**v) Commutative Law for Addition**



1. **Multiplication Properties**

**i) Closure Law**



**ii) Associative Law**



**iii) Multiplicative Identity**

 such that



1 is called the multiplicative identity of real numbers.

**iv) Multiplicative Inverse**

 such that



**v) Commutative Law of multiplication**



1. **Distributive Properties**



 (Left Distributive Prop.).

(Right Distributive Prop.).

1. **Equality Properties**

*Equality of numbers denoted by “=”*

*possesses the following properties: -*

**i) Reflexive property**



**ii) Symmetric Property**



**iii) Transitive Property**



**iv) Additive Property**





**v) Multiplicative Property**







**vi) Cancellation Property w.r.t addition:**





**vii) Cancellation Property w.r.t**

**Multiplication:**





1. **Inequality Properties**

**i)** **Trichotomy Property**





**ii) Transitive Property**



● 

● 

**iii) Additive Property:**



● 

● 

● 

● 

**iv) Multiplicative Properties:**

**(a).** 

● 

● 

**(b).** 

● 

● 

**(c).**  are all

positive

● .

● 

♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦

**Exercise 1.1**

**Q. No. 1:**

**Which of the following sets have closure property w.r.t addition and multiplication?**

**i)**  **ii)** 

**iii)**  **iv)**

**Sol.**

**(i)** 



Set is closed w.r.t. +.



Set is closed w.r.t ×.

**(ii)** 

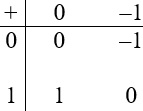


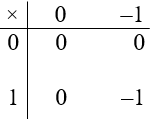
Set is not closed w.r.t. +.



Set is closed w.r.t. ×.

**(iii)** 



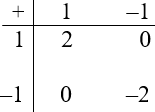


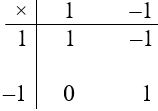
It is clear from the adjoining tables that the set

is not closed w.r.t. “+” and closed w.r.t. “×”

because every element appeared in table is the

element of set.

**(iv)** 



It is clear from the adjoining tables that the set

is not closed w.r.t. “+” and “×” as well

because elements appeared in table are not all

the elements of set.

**Q. No. 2:**

**Name the property used in the following equations?**

i) 

ii) 

iii) 

iv)  v) 

vi)  vii) 

viii) 

ix) 

x) 

xi) 

xii) 

**Sol.**





*Commutative prop. w. r. t addition.*





*Associative prop. w.r.t addition*



*Associative prop. w.r.t. addition*

1. 



*Additive identity prop.*









*Multiplicative identity prop.*



Additive inverse prop.





*Commutative prop. w.r.t. multiplication*





*Left distributive prop.*





*Right distributive prop.*

**xi)**





Associative prop. w.r.t. multiplication.

**xii)**



Left distributive prop.

**Q. No. 3:**

**Name the property used in the following inequalities:**

i)  ii) 

iii)  iv) 

v) .

**Sol.**







Additive property.







Multiplicative prop.







Additive prop.





Multiplying by – 1 on both sides





*Multiplicative prop.*









*Multiplicative prop.*

**Q. No. 4:** **Prove the following rules of addition.**

**Solution:**



L.H.S.







=R.H.S.



L.H.S.=













=R.H.S.

**Q. No. 5:**

**Prove that** 

**Solution:**















=R.H.S.

**Q. No. 6:**

**Simplify by justifying each step.**

**Solution:**



 *Multiplicative Property*

* Right Distributive Law*

* Multiplicative Inverse*

* Multiplicative Identity*



 *Golden rule of fraction*

 *Right distributive prop*

 *Multiplicative prop.*

*Associative prop. w.r.t.×.*

 *Multiplicative inverse*

 *Multiplicative Identity*

 *Additive Property*



**iii)**



 *Golden rule of fraction.*

 *Right distributive prop.*

*Multi. inverse prop.*

*Multiplicative identity prop*

♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦

**COMPLEX NUMBERS:**

An expression of the form, where *a* and *b*

are real numbers and , is called complex

number. It is usually denoted by z. i.e. 

where *a* is called real and *b* is called imaginary

part of the complex number z and may be

denoted by Re(z) and Im(z) respectively. A

complex number may also be represented as an

ordered pair of a real numbers.

For example

4, 1, etc. are complex numbers.

|  |  |  |  |
| --- | --- | --- | --- |
| Complex Numbers | Ordered Pair Form | Real Part | Imaginary Part |
|  |  |  |  |
|  |  | 2 | 7 |
|  |  | 3 |  |
|  |  | 100 | 1 |
|  |  | 0 | 1 |
|  |  | 8 | 0 |
|  |  | 3 | 1 |

**Note:** Every Real number is a Complex number but every Complex number is not a Real Number.

**OPERATIONS ON COMPLEX NUMBERS:**

1. **Addition** for two complex numbers and is defined as



1. **Subtraction** for two complex numbers andis defined

as 



**3. Multiplication** of two complex

numbersand  is

defined as



**4. Division** of two complex numbers

and  where

is







**Find the multiplicative inverse of a non-**

**zero complex number?**

Let  be a complex number if 

then  is its multiplicative inverse.











**MODULUS OF A COMPLEX NUMBER:**

Modulus of a complex number isdenoted as  or mod (*z)* and defined as where 

Sometimes, is called absolute value of z.

Note that for any complex number.

For example,

if  then



**PROPERTIES OF MODULUS**

1. 
2. 
3.  and 
4. ,
5. ,
6. ,
7. ,
8. ,
9. .

**CONJUGATE OF COMPLEX NUMBER**

Conjugate of a complex number  is

denoted by and defined as 

For example,

1. 
2. 
3. 
4. 

**PROPERTIES OF CONJUGATE**

1. 
2. 
3. 
4.  if and only if *z* is purely real
5.  if and only if *z* is purely imaginary
6. and 
7. 
8. 
9. 

**Exercise 1.2**

**Q. No. 1:**

**Verify the addition properties of complex numbers.**

**Solution:**

Let 

Where  and 

1. **Closure property**

If  then 



1. **Associative property**

If 

then 













1. **Identity Property**

 is additive identity for any .





1. **Inverse Property**

For any there exist

its additive inverse such that





1. **Commutative property**

If then 









**Q. No. 2:**

**Verify the multiplication properties of complex numbers.**

**Solution:**

Let ,

where  and 

1. **Closure property**

If  then 



1. **Associative property**

If 

then 



















Hence 

1. **Identity Property**

 is multiplicative identity for any









1. **Inverse Property**

For all their exist its multiplicative inverse  such that









1. **Commutative property**

If then 











Hence



**Q. No. 3:**

**Verify the distributive law of complex numbers.**



**Solution:**

L.H.S. 







R.H.S. 





Hence verified.

**Q. No. 4:**

**Simplify the following**

i)  ii) 

iii)  iv) 

**Solution:**





















































**Q. No. 5:**

**Write in term of **

i)  ii) 

iii)  iv) 

**Solution:**













**iii)**







**iv)**











**Q. No. 6:**

**Simplify **









**Q. No. 7:**

**Simplify **







**Q. No. 8:**

**Simplify **



**Q. No. 9:**

**Simplify **



**Q. No. 10:**

**Simplify **



**Q. No. 11:**

**Simplify **



**Q. No. 12:**

**Simplify **





**Q. No. 13:**

**: Prove that the sum as well as the product of any two conjunction complex numbers is a real number.**

**Solution:**





**Sum**







**Product**





**Q. No. 14:: Find the multiplicative inverse of each of the following numbers.**

i)  ii) 

iii) 

**Solution:**

**(i)** Let 









**(ii)**

Let 







**(iii)**



The multiplicative inverse of 



The multiplicative inverse of 







**Q. No. 15:** **Factorize of the following.**

i)  ii) 

iii) 

**Solution:**

**(i)**











**(ii)**











**(iii)**











**Q. No. 16:**

**Separate into real and imaginary parts.**

i)  ii) 

iii) 

**Solution:**















Real part

Imaginary part

**(ii)**

















Real part

Imaginary part

**(iii)**















Real part

Imaginary part

♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦♦